D-18174

AN ANALYTICAL STUDY OF THE GUST-LOAD ALLEVIATION OBTAINED BY MEANS OF A NORMAL-ACCELERATION-CONTROLLED FLAP

A Thesis

Presented to

the Faculty of the Department of Engineering

University of Virginia

In Partial Fulfillment

of the Requirements for the Degree

Master of Aeronautical Engineering

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by

Martin H. Schindler

June 1952

N71 72375
(ACCESSION NUMBER) (THRU)
61 none
(PAGES) (CODE)
TMX 67034
(NASA CR OR TMX OR AD NUMBER) (CATEGORY)
FACILITY FORM 602

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VIRGINIA

1952

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CHAPTER I

STATEMENT OF PROBLEM

The loads on an airplane flying in turbulent air are proportional to the gust intensity and tend to increase with airplane flight speed, slope of the wing lift curve, and airplane gross weight. As airplanes become bigger and faster, designing the airplane for the gust load condition becomes more important. Furthermore, it is desirable for these larger and faster airplanes to operate in all weather conditions. Therefore, the study of gust alleviation has been of interest for a number of years; without using gust alleviation devices, it might be necessary to increase the structural strength of these airplanes to permit them to operate in all weather conditions, or to decrease their speed while operating in rough air.

A device that is controlled by some effect of the gust to reduce the aerodynamic loads caused by the gust can potentially be used as a gust alleviator. A flap actuated by a servomechanism which is, in turn, controlled by some characteristic of the gust is such a device. This type of gust alleviator has promise of being effective due to recent improvements made in servomechanisms. However, few results of systematic

studies of how this type of gust alleviator affects the characteristics of the airplane are available. Therefore, an analytical investigation was made of a flap controlled by normal acceleration used as a gust alleviator in order to determine its effects on the airplane motions and the aerodynamic loads caused by the gust.

For the purpose of this analytical study, the amount of gust alleviation is considered to be the difference, as a fraction of the unalleviated normal-acceleration increment, between the normal-acceleration increment caused by the gust on the airplane without the alleviation system and that caused on the airplane with the alleviation system. The effects of airplane speed, gust intensity, flap-servo sensitivity, and airplane longitudinal static stability were studied in order to find the influence of these airplane parameters on the amount of gust alleviation obtainable.

CHAPTER II

PREVIOUS AND RELATED STUDIES

The principal method of gust alleviation is to reduce the effectiveness of the wing to produce lift in a gust. This can be done by a reduction of the effective slope of the airplane lift curve or a change in the lift-mass ratio. The effective static slope of the airplane lift curve can be changed aerodynamically by changing the wing aspect ratio or the angle of wing sweep. The effective static slope of the airplane lift curve can also be changed aerodynamically by destroying part of the air flow over the wing by use of a fixed spoiler. The effective dynamic slope of the airplane lift curve can be changed aeromechanically by having a servomechanism operate a flap or spoiler in proportion to some effect of the gust. The dynamic slope of the airplane lift curve in a gust can be changed aeroelastically by having the wings twist to obtain washout or by having the wing bend to spill some of the aerodynamic load.

Pierce[1] and Reisert[2] have conducted an extensive study of swept wings in the Langley Gust Tunnel. This aerodynamic device was investigated for use in alleviating gust loads. This study showed that the amount of maximum normal acceleration

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[1] Harold B. Pierce, Tests of a 45° Sweptback-Wing Model in the Langley Gust Tunnel. N.A.C.A. T.N. 1528, 1948, P. 1; _____, Gust-Tunnel Investigation of a 45° Sweptforward-Wing Model. N.A.C.A. T.N. 1717, 1948, P.1; _____, Gust-Tunnel Investigation of a Wing Model with Semichord Line Swept Back 60°. N.A.C.A. T.N. 2204, 1950, P. 1.

[2] Thomas D. Reisert, Gust-Tunnel Investigation of a Wing Model with Semichord Line Swept Back 30°. N.A.C.A. T.N. 1794, 1949, P.1.

increment for the swept wing may be assumed to equal the equivalent straight wing maximum normal acceleration increment times the cosine of the angle of sweep. This study indicated that the gradual penetration of the swept wing into the gust ~~effects~~ reduces the maximum normal acceleration increment.

Mickleboro[3] has evaluated the use of a fixed spoiler, an aerodynamic gust alleviator, to alleviate gust loads from gust tunnel tests. These tests indicated that in a sharp edge gust the fixed spoiler gave no alleviation, but that in a gradient gust gave thirty percent normal acceleration increment alleviation. Even though a spoiler properly placed will ~~reduce~~ reduce the lift curve slope sixty-five percent, causing a possible corresponding amount of load alleviation, it did not seem to; probably due to aerodynamic lag in the system. Mickleboro also pointed out that spoilers could probably only be used to reduce positive loads.

Shuffleberger[4] has investigated the alleviation of airplane normal acceleration by use of a torsionally flexible wing. This aeroelastic gust alleviator was a wing with the torsion axis ahead of the locus of the section aerodynamic

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[3] Harry C. Mickleboro, Evaluation of a Fixed Spoiler as a Gust Alleviator. N.A.C.A. T.N. 1753, 1948, P.1.

[4] C. C. Shuffleberger, Tests of a Gust-Alleviating Wing in the Gust Tunnel. N.A.C.A. T.N. 802, 1941, P.1.

centers. This study indicated that such a device has five percent alleviation in a sharp edge gust and seventeen percent in a gradient gust. This investigation also indicated that this type of alleviator has a tendency to increase the longitudinal stability of the model in a gust.

Reisert[5] has conducted a study of a sweptback wing which was free to bend in the gust tunnel. This combination of an aerodynamic and aeroelastic gust alleviator was investigated for reducing wing gust loads. This investigation indicated that the over-all load alleviation caused by this device was small due to the severe adverse pitching motion it caused. However, it was found that the net reduction in bending moment may be appreciable by using such a device.

Donely and Shufflebarger[6] have conducted tests on a gust-alleviating full span flap, an aeromechanical gust alleviator, in the gust tunnel. This flap was over balanced so that it moved to reduce the airplane wing loads whenever the plane underwent normal displacement. These tests indicated that thirty-nine percent gust load alleviation was obtainable

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[5] Thomas D. Reisert, Gust-Tunnel Investigation of a Flexible Wing Model with Semichord Line Sweptback 45°. N.A.C.A. T.N. 1959, 1949, P.1.

[6] Philip Donely and C. C. Shufflebarger, Tests of a Gust Alleviating Flap in the Gust Tunnel. N.A.C.A. T.N. 745, 1940, P.1.

by the system in a gradient gust, but only three percent gust load alleviation was obtainable by the system in a sharp edge gust. These tests also show negligible changes in pitch when using this device.

Phillips and Kraft[7] carried out an analytical analysis of an angle of attack controlled flap linked to the elevator of the airplane for use as a gust alleviator. This aeromechanical gust alleviator was investigated for possibilities of using it to improve the riding comfort of an airplane in flight through atmospheric turbulence. This analysis indicated that to completely eliminate the pitching moment, pitch, and normal acceleration, caused by gusts and thus increase the smoothness of flight in gusts, it is necessary to use full span flaps linked to the elevator as the gust alleviator.

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[7] William H. Phillips and Christopher C. Kraft, Jr., Theoretical Study of Means for Increasing the Smoothness of Flight Through Rough Air. NACA TN , 1951

CHAPTER III

THE PRESENT INVESTIGATION

The purpose of the present investigation is to study the effects on an airplane of a normal acceleration controlled flap used to reduce the gust loads on an airplane. This reduction in gust loads is accomplished by reducing the gust load factor. However, the flap deflection to do this causes a high pitching moment on the airplane.

This high pitching moment may cause adverse effects on the airplane. The pitching moment may cause the airplane to pitch so as to reduce or cancel the effectiveness of the flap in alleviating the gust loads. This pitch may become so severe that the performance of the airplane will be adversely affected by the flap deflection. The flap deflection may change the pitching moment of the airplane so that the airplane will be unstable.

The effects of several of the airplane parameters and characteristics of the alleviation system that effect the amount of this pitching moment produced by the alleviator and the resultant pitch are studied in this investigation. The alleviation that is produced by the flap is directly proportional to the flap deflection when all other variables are neglected. Flap deflection, as stated before, causes a large pitching moment. A

change of forward velocity changes the dynamic pressure which effects the aerodynamic loads and moments on an airplane. The static margin of longitudinal stability effects the amount of pitch a given pitching moment will cause. Variation of the natural frequency of the flap servomechanism effects the amount of flap deflection and thus the flap effectiveness and the resultant pitching moment. Thus, for this present study the effects on an airplane of a normal acceleration controlled gust alleviating flap are investigated when the flap servo sensitivity, airplane speed, and airplane longitudinal stability are varied.

CHAPTER IV

METHOD

For this investigation it is necessary to solve the three degree of freedom equations containing terms for normal acceleration, airplane pitch, and flap deflection which represents the airplane and alleviation system. These equations are solved numerically by a method, given by Mazelsky and Diederich[1] and using the Bell digital computer. By this method the equations of motion and the DuHamel's integral's they contain are put into numerical recurrence equation form which are then denoted by matrix notation. Other methods of solution of these equations were considered but they were not considered practical. To solve these equations exactly the method of operators could be employed but this would require as part of the solution obtaining the roots of a thirty degree polynomial. To solve the equations manually by graphical means is too tedious because of the large number of DuHamel's integrals involved. These equations could be solved by the Reeves antalog computer, which would solve the equations automatically by graphical means, however, the one now available cannot handle all the Duhamel's integrals in the equations.

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[1] Bernard Mazelsky and Franklin W. Diederich, A Method of Determining the Effect of Airplane Stability on the Gust Load Factor.

The following equations of motion were set up to include the three degrees of freedom involving normal acceleration, pitch, and flap deflection:

$$(1)\ \Delta n(s) = \Delta n_w(s) + \Delta n_t(s)$$

$$(2)\ D^2\theta = \Delta n_w(s)\cdot K_{25} - \Delta n_t(s)\cdot K_{26} + K_e \int_0^{s_1}\int_0^{s_1} D^2\delta$$

$$(3)\ D^2\delta = K_f \Delta n(s) + K_g \int_0^{s_1}\int_0^{s_1} D^2\delta + K_h \int_0^{s_1} D^2\delta$$

where

$$\Delta n_w(s) = K_1 \int_0^{s_1} C_{L_{g_w}}(s_1-s)\, D\frac{u}{u_{MAX}}\, ds - K_4 \int_0^{s_1} C_{L_{\alpha_w}}(s_1-s)\,\Delta n(s)\, ds$$
$$+ K_7 \int_0^{s_1} C_{L_{\alpha_w}}(s_1-s)\, D\theta\, ds - K_{10} \int_0^{s_1} C_{L_{\alpha_w}}(s_1-s)\, D^2\theta\, ds$$
$$+ K_a \int_0^{s_1} C_{L_{\alpha_w}}(s_1-s)\, D\delta\, ds + K_b \int_0^{s_1} C_{L_{\alpha_w}}(s_1-s)\, D^2\delta\, ds$$

and

$$\Delta n_t(s) = K_2 \int_0^{s_1} C_{L_{g_t}}(s_1-s)\, D\frac{u}{u_{MAX}}\, ds + K_3 \int_0^{s_1} C_{L_{\epsilon_g}}(s_1-s)\, D\frac{u}{u_{MAX}}\, ds$$
$$- K_5 \int_0^{s_1} C_{L_{\alpha_t}}(s_1-s)\,\Delta n(s)\, ds - K_6 \int_0^{s_1} C_{L_{\epsilon_g}}(s_1-s)\,\Delta n(s)\, ds$$
$$+ K_8 \int_0^{s_1} C_{L_{\alpha_t}}(s_1-s)\, D\theta\, ds + K_9 \int_0^{s_1} C_{L_{\epsilon_g}}(s_1-s)\, D\theta\, ds$$
$$+ K_{11} \int_0^{s_1} C_{L_{\alpha_t}}(s_1-s)\, D^2\theta\, ds - K_{12} \int_0^{s_1} C_{L_{\epsilon_g}}(s_1-s)\, D^2\theta\, ds$$

The algebraic expressions for the constants used in these equations are given in Appendix A.

These equations contain terms contributed by the gust, normal motion of the airplane, the pitch of the airplane, and the flap deflection. The term containing the constant K_1 gives the lift on the wing due to the gust. The lift on the tail caused by the gust is given by the term containing K_2. The downwash on the tail caused by the wing penetrating the gust is given by the term containing K_3. The lift on the wing due to airplane pitch is given by the term containing K_7; that on the tail, by the term containing K8. The downwash on the tail caused by the wing when the airplane pitches is given by the term with K_9. The lift on the wing due to the rate of airplane pitch is given by the term containing K_{10}; that on the tail, by the term containing K_{11}. The downwash on the tail caused by the wing due to the rate of pitch is given by the term containing K_{12}. The lift on the wing caused by flap deflection is given by the term containing K_a. The lift on the wing caused by rate of flap deflection is given by the term containing K_b. In order to simplify the equations the downwash caused by flap deflection and rate of flap deflection is neglected.

The third equation of motion is obtained by setting the summation of flap hinge moments equal to the inertia moment due to flap deflection. The aerodynamic flap hinge moments are assumed equal to zero. Therefore, the only flap hinge moments acting are those due to the servomechanism. These hinge moments are the driving hinge moment, which is proportional to the normal acceleration; the hinge moment due to the servo spring, which depends on the mass of the flap and its linkage system and the natural mechanical frequency of the flap servo system; and the hinge moment due to the damping in the system, which depends on the spring constant, mass of the flap and its linkage system, and the amount of critical damping involved. Figure 1 shows a schematic diagram of the flap servomechanism. From it it can be seen that the equation for flap hinge moments is simply an equation of forces vibration with viscous damping.

In Appendix B will be found the numerical integrating formulas needed to solve these equations of motion. By using these integrating formulas on the equations of motion and collecting like terms, the equation can be put into recurrence equation form. The recurrence equation form is expressed by matrix notation as shown in Appendix C.

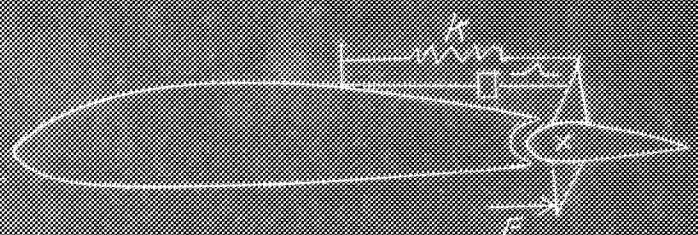

Figure 1—Schematic diagram of flap servo mechanism.

CHAPTER V

CONDITIONS

As can be seen by looking at the equations of motion given in Chapter IV and the recurrence form of the equations given in Appendix C, the gust contributes the forcing functions to these equations. Therefore, it is necessary to decide what type of gust shape and intensity will be used in solving these equations. Donely[1] in his summary report points out that the most probable gust encountered by an airplane in rough air flight is a single ten chord gradient gust. A gust velocity of thirty feet per second is the usual design gust velocity for large transport airplanes. Therefore, in this investigation the gust will be assumed to be a single ten chord gradient gust with a gust velocity of thirty feet per second.

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1. Philip Donely, Summary of Information Relating to Gust Loads on Airplanes. N.A.C.A. TN 1976, 1949, P. 12

The unsteady lift functions due to gust penetration and angle of attack change are obtained from the work of Jones[2] on this subject. His values for two-dimensional flow are used. In Appendix D are found the values of unsteady lift due to an angle of attack change used in this investigation. The values of unsteady lift functions due to gust penetration for a gradient gust used in this study is found in Appendix F.

When using the numerical method ~~that is used in~~ of this present investigation to solve the equations of motion, it is necessary to decide what the interval of integration is to be in the recurrence equations and integrating formulas. For numerical integrating formulas to be accurate it is necessary that there be approximately four intervals per half cycle of the curve that is to be integrated. It is assumed that the unsteady lift due to angle of attack change of the wing is significant for nine chord lengths of travel after an angle of attack change. ~~Therefore~~ Thus, the interval of integration determines the number of terms in the recurrence equations. Therefore, it is necessary to compromise as to the value of this interval of integration for accurate use of the integrating formulas and for as few as possible terms in the recurrence equations. For this investigation using a ten chord gradient gust, an interval of integration of one and half chord lengths of travel was found

[2] Robert T. Jones, The Unsteady Lift of a Wing of Finite Aspect Ratio. NACA Report 681, 1940, Figure 9.

to be satisfactory.

The characteristics of a hypothetical transport type of airplane are used in the solution of these equations. The values of the hypothetical parameters of an airplane are given in Appendix E. A skeleton diagram of the hypothetical airplane is given in Figure 2.

Since the solution of the numerical equations does not show how the various airplane parameters effect the alleviation obtainable, it is necessary to solve the equations several times while the various airplane parameters are varied. This is done by assuming a standard condition and then varying the airplane parameters one at a time. The standard condition is for a forward velocity of 300 mph, a static longitudinal stability margin of -0.35, a critically damped alleviation system, a natural frequency of the alleviation system of three cycles per second, and one hundred percent static alleviation. Forward speed, static stability, natural frequency, and static alleviation are then varied separately from these values.

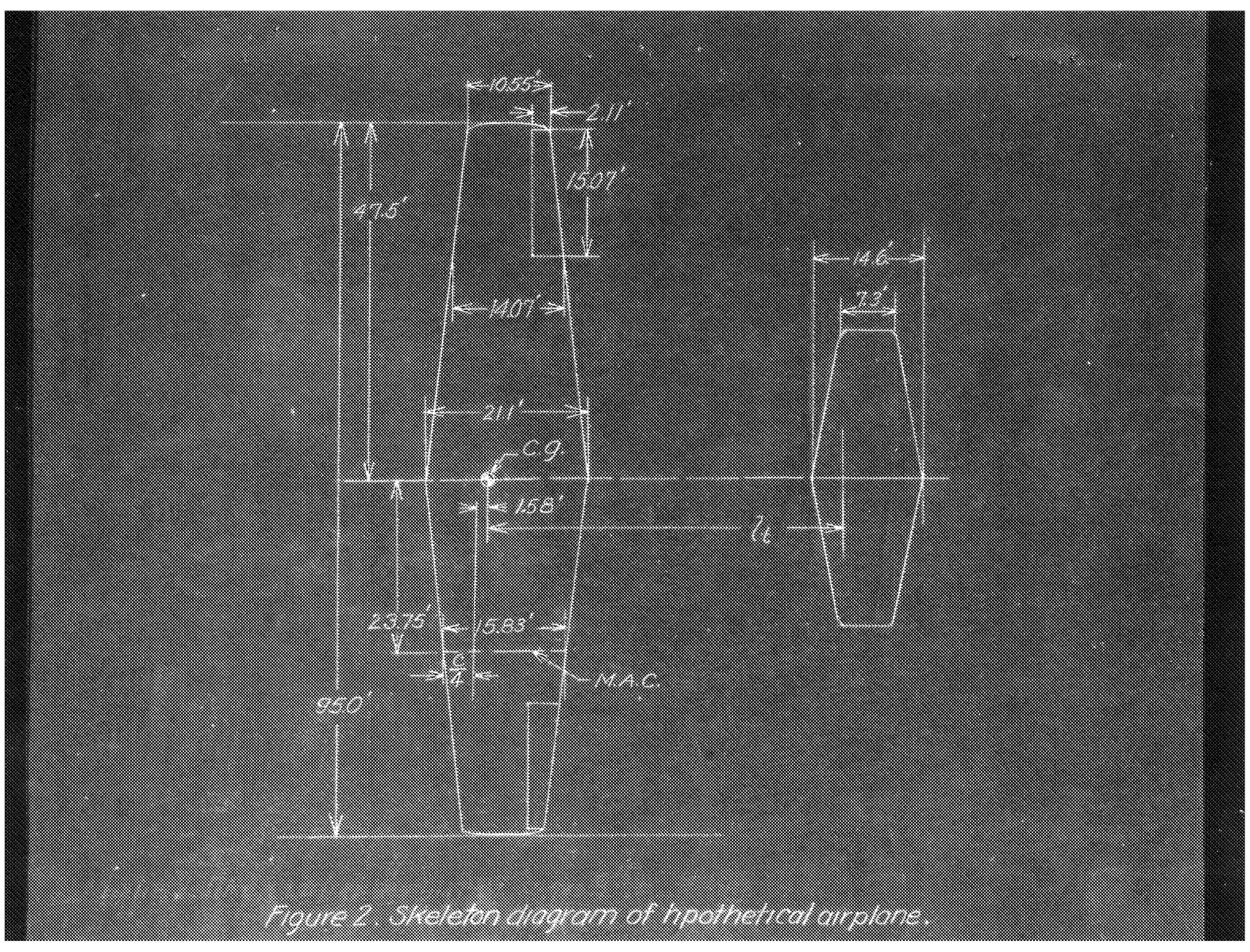

Figure 2. Skeleton diagram of hypothetical airplane.

CHAPTER VI

RESULTS

The solution of the equations of motion were made to obtain time histories of normal acceleration increment, total airplane pitch, which is the summation of flight path angle and angle of attack, and the flap deflection. Figure 3 shows some typical time histories as affected by variations in the static alleviation of the system.

Another way of representing the effects of varying static alleviation is shown in figure 4. This figure shows that the dynamic alleviation and the flap deflection vary approximately as the square root of the static alleviation. Both the dynamic alleviation and flap deflection tend to reach an asymptote as the static alleviation increases. It can also be seen that the pitch has a negligible effect on the maximum positive load factor. The lag in the system tends to decrease as the static alleviation increases.

The effects of varying the airplane forward velocity are shown in figure 5. The dynamic alleviation and maximum flap deflection for one hundred percent static alleviation increase as the forward velocity increases. The pitch seems to have

(a) Triangular gust with a gradient distance of 10 chords.

Figure 3.- Time histories of normal acceleration, pitch, and flap deflection during penetration of gust for various amounts of static elevation.

(b) Gust with a 42-chord gradient distance.

Figure 3.- Concluded.

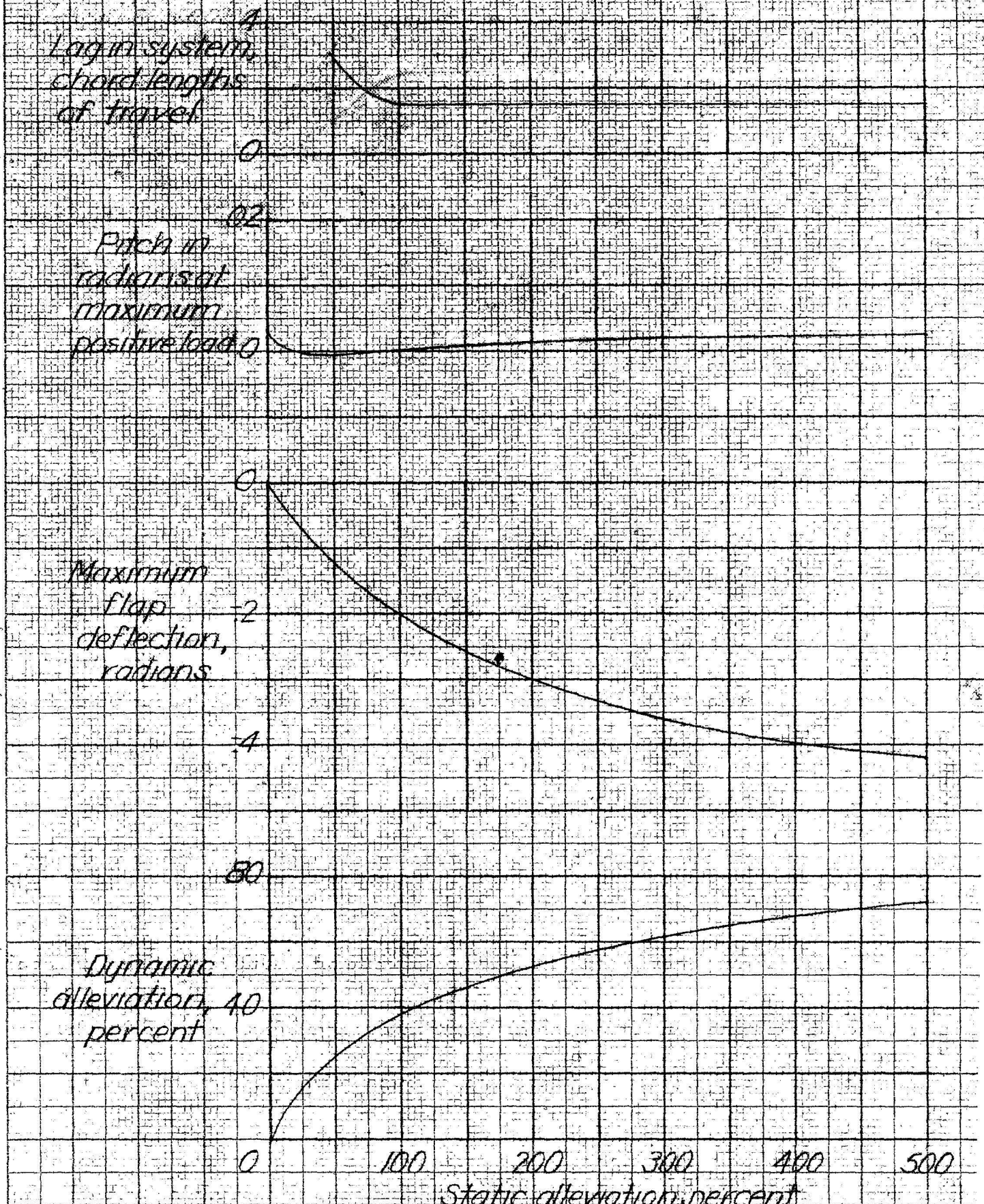

Figure 4.- Effect of varying static alleviation of airplane in triangular gust with 10-chord gradient distance.

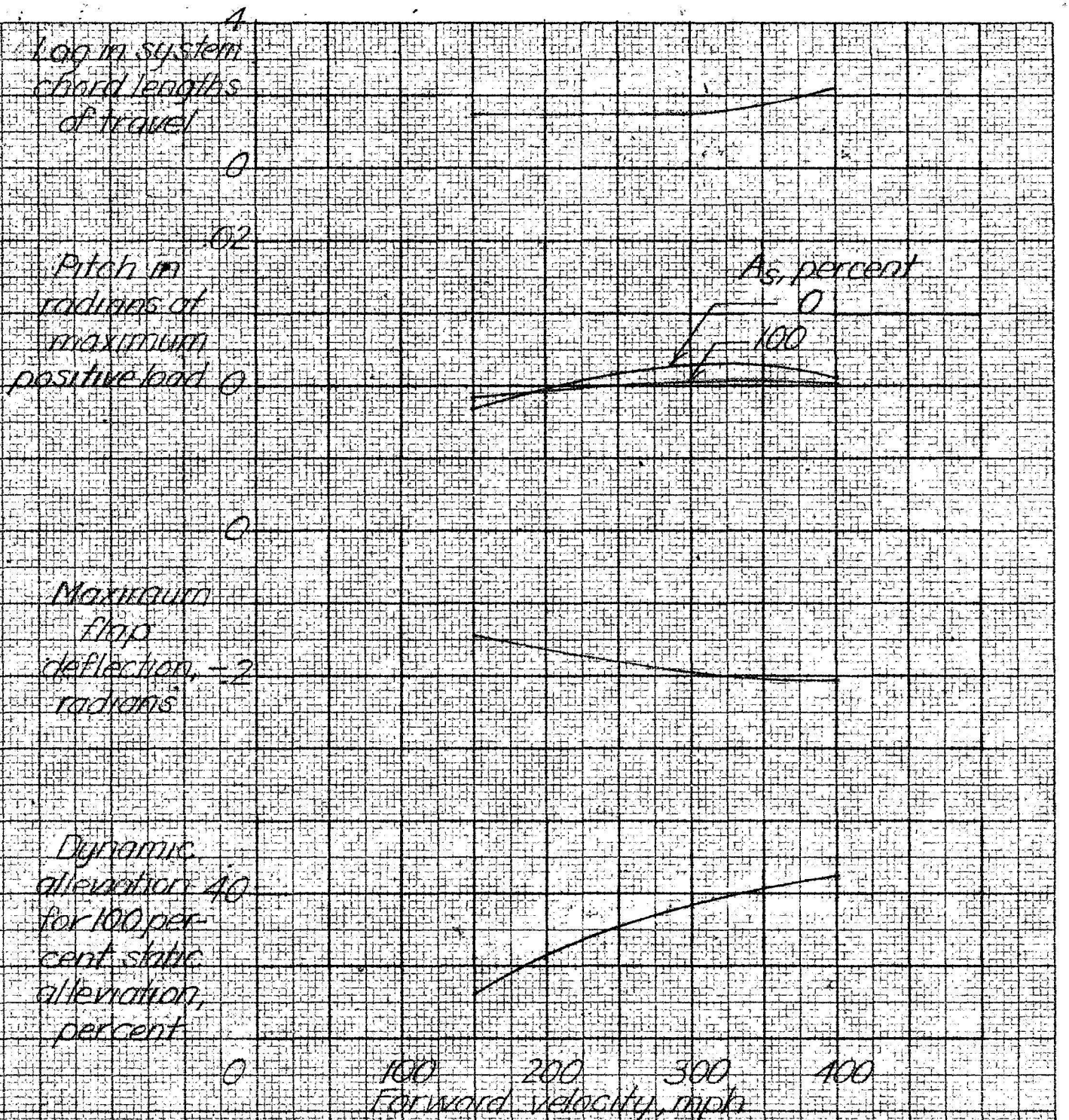

Figure 5.- Effect of varying forward velocity of airplane in a triangular gust with 10-chord gradient distance

little effect on the maximum positive load factor as the forward velocity is varied. A change in the forward velocity does not seem to effect the lag in the system.

The effect of varying the static margin of longitudinal stability is shown in figure 6. This indicates that as the static stability is increased the dynamic alleviation has little variation while maximum flap deflection is slightly decreased. It seems that the pitch has little effect on the maximum positive load factor. The lag in the system tends to decrease as the static margin of longitudinal stability is increased.

In figure 7 are shown the effects of changing the natural frequency of the flap-servomechanism. The frequency is varied by assuming different values for the spring constant. As the natural frequency increases, the dynamic alleviation and maximum flap deflection tend to increase, and the lag in the system tends to decrease. The pitch seems to have little effect on the maximum positive load factor.

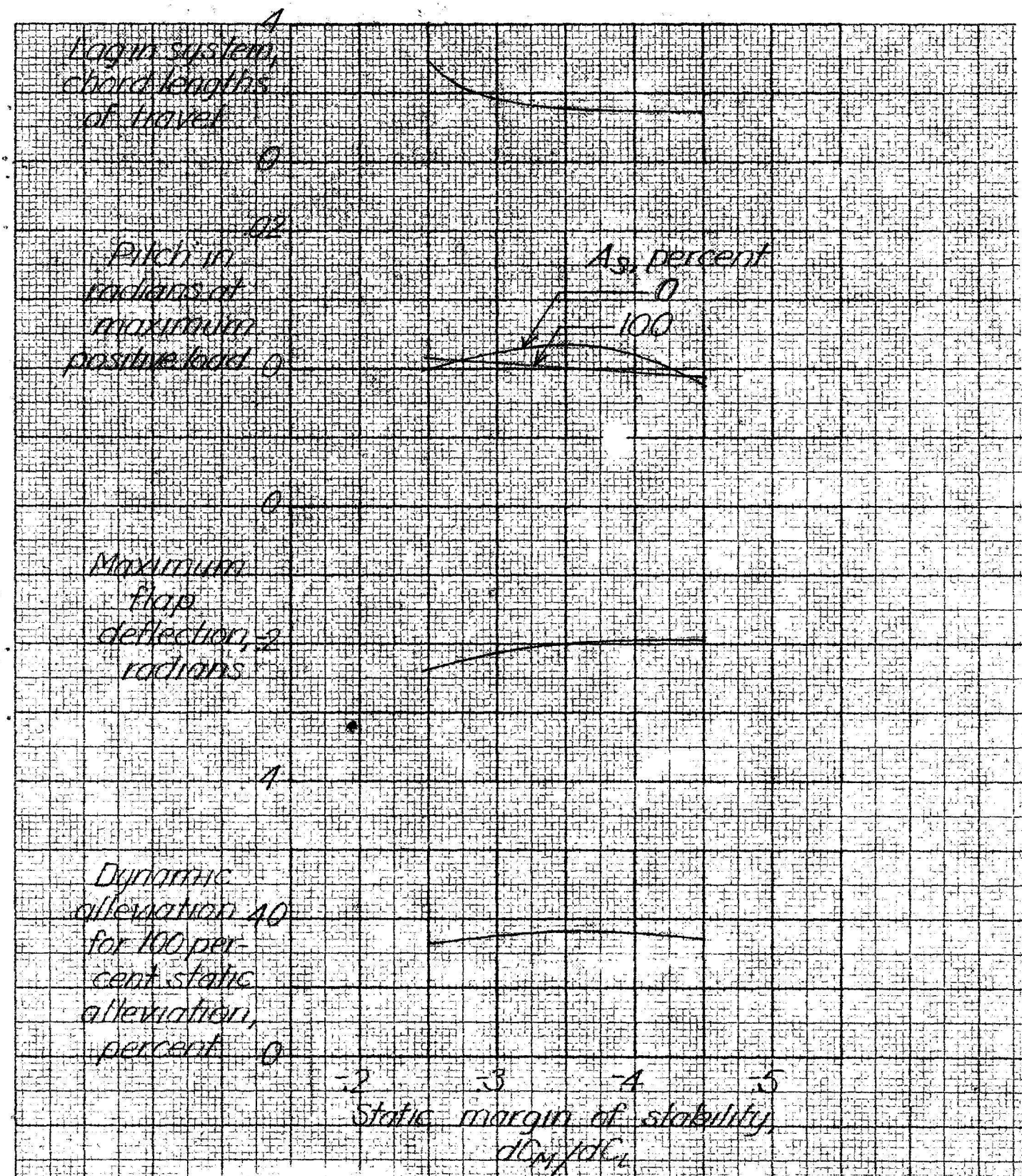

Figure 6.—Effect of varying static margin of airplane in triangular gust with 10-chord gradient distance.

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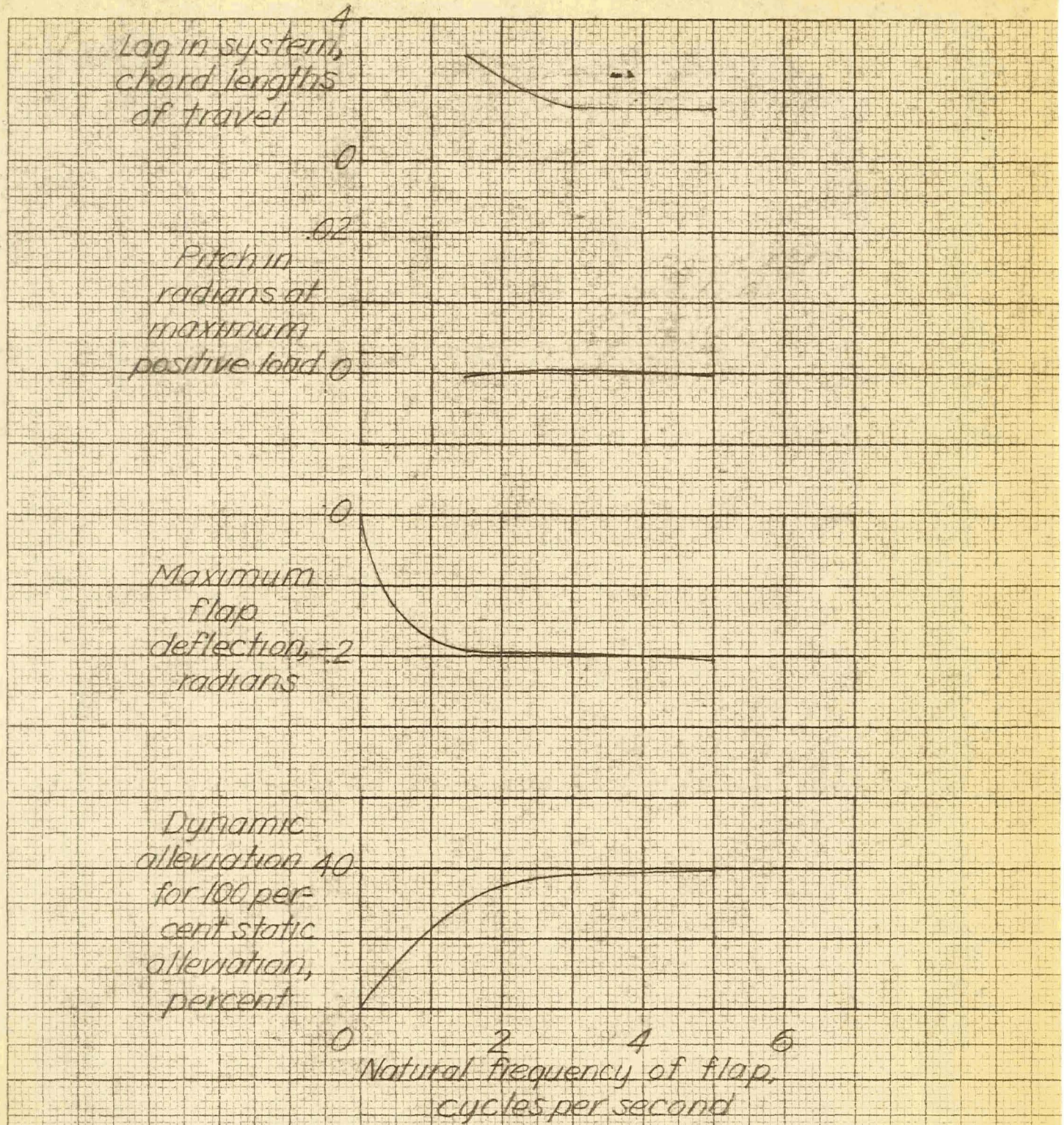

Figure 7.– Effect of varying natural frequency of flap of airplane in triangular gust with 10-chord gradient distance.

CHAPTER VII

DISCUSSION OF RESULTS

The purpose of this investigation is to determine whether or not a normal acceleration controlled flap can be used effectively to alleviate gust loads on large high speed airplanes. To do this it is necessary to see how much alleviation is obtainable, by this device, if the pitching moment caused by the flap causes adverse effects, and what are the most favorable airplane characteristics for using this gust alleviator.

A normal acceleration controlled gust alleviating flap cannot obtain more than one hundred percent gust load alleviation, since for this device to operate, it is necessary that a normal acceleration increment act on the airplane. Therefore, as the static alleviation becomes infinite, the dynamic alleviation approaches one hundred percent. To have effective load alleviation, it is not necessary to have one hundred percent normal acceleration increment alleviation; and, therefore, a normal acceleration controlled gust alleviating flap can be used effectively to reduce gust loads.

In figure 4 it is shown that as the static alleviation increases, the dynamic alleviation and flap deflection increase and the lag of the system decreases. This means that as the static alleviation is increased, the effectiveness of the alleviation system is increased. This is caused by the fact that increasing the static alleviation increases the amount of flap deflection per g of normal acceleration increment which in turn affects the flap driving hinge moment as shown in Appendix G.

Changing the airplane forward speed changes the static alleviation since the static deflection of the alleviator per g of normal accdleration increment is kept constant while the dynamic pressure varies. Thus, when the static alleviation is 100 percent for an airplane forward speed of 300 mph, it will be 178 percent for a speed of 400 mph. Likewise for an airplane forward speed of 150 mph, the static alleviation decreases to 25 percent. By comparing figures four and five, it can be seen that the dynamic alleviation obtained at these forward speeds and that obtained at the corresponding values of static alleviation are approximately the same. If it is desired to keep the alleviation constant as the airplane forward speed is changed,

it would be necessary to vary the static deflection per g approximately in proportion to the change in dynamic pressure.

The flap deflection increases non-linear with an increase in forward speed, as shown in figure five, since the alleviated normal acceleration increment, which causes the flap deflection, is non-linear with airplane speed. However, the unalleviated normal acceleration increment varies linearly with airplane forward speed and so might lead to the belief that the flap deflection does also. The variation in flap deflection caused by forward speed changes is not as large as the variation in alleviation caused by forward speed changes due to the flap load varying directly with dynamic pressure.

Previous experience has indicated that changing the gust velocity will have a negligible affect on the percent of normal acceleration increment alleviation. This can be seen by an examination of the normal acceleration controlled gust alleviating flap under static conditions. This examination indicates that the normal acceleration increment due to a gust, the flap deflection, and the amount of normal acceleration reduction due to the flap deflection vary linearly with the gust velocity. Thus, the effect of gust velocity on the percent of normal acceleration increment alleviation can be disregarded.

As the natural frequency is changed by changing the spring constant, the driving moment has to be changed to keep the static deflection per g constant. For this condition and for a constant static alleviation, the response of the gust alleviator to a normal acceleration increment approaches the static case. Due to this change of response of the alleviator, the dynamic alleviation and the flap deflection increase as the natural frequency increases as shown in figure 7. Also, the lag in the system tends to decrease from an infinite amount for zero natural frequency to a small finite amount for a higher frequency. Therefore, it would seem that the higher the natural frequency the more effective a normal acceleration controlled flap is as a gust alleviator. However, the higher the natural frequency the more difficult it is design a light weight servo-system. Also, past some value of natural frequency, the rate of increase in alleviation as the frequency increases becomes negligible. These two factors limit the maximum value of natural frequency that can be used effectively.

It might be suspected that some gust shape could cause the gust alleviator to resonate so that the alleviator would increase the loads instead of decreasing them. This

could probably happen when the gust gradient distance corresponds to the distance the alleviator needs to complete one cycle of deflection. In a majority of the conditions analyzed these distances were nearly the same. The results show that the alleviator will probably not resonate due to gust gradient distance so as to cause adverse loads, if it is critically damped.

Figure 6 shows the effect of varying the static margin of longitudinal stability which was done by changing the tail length and thus the tail volume. This method of varying the static margin seems to have little effect on gust load alleviation. Further work indicates that when the static margin is increased by moving the center of gravity forward the gust load alleviation decreases slightly. As the static stability is increased, the maximum normal acceleration increment due to a gust decreases slightly. This causes the flap deflection and thus the associated load reduction to decrease slightly. However, these slight changes have negligible effect on the dynamic alleviation.

The fact that variation of the static margin has little effect on dynamic alleviation may lead to the belief that the pitch due to flap deflection has little effect on alleviation. Figure 3 shows that the pitch due to flap

deflection is negligible for the first thirty chords of gust penetration, but after this point, it is appreciable. Figure 3a shows that for a single gust with a short gradient distance, the normal acceleration increment is small where the change in pitch due to flap deflection is large. Therefore, for this type of gust, the effect of the change in pitch due to flap deflection can be assumed negligible. Figure 3b shows that for a single gust with a long gradient distance, the normal acceleration increment is very close to a maximum value where the change in pitch due to flap deflection is large. However, this figure also shows that the change in pitch is in a direction to help the alleviation of an airplane in the gust with the long gradient distance. Thus, the investigation shows that for a single gust, the pitch due to flap deflection is negligible for the short gradient distance, but not for the long gradient distance; however, the investigation shows that this pitch does not cause adverse effects on the alleviation in a single gust.

Figure 3 also shows that once the airplane has past through a gust, the pitch due to flap deflection can be from 30 to 50 percent of the total airplane pitch in the alleviated cases. This means that the alleviator can adversely effect the subsequent airplane operations. It is possible that the pitch due to flap deflection in the first

gust of a series of gusts can produce adverse effects on the gust load alleviation in the subsequent gusts. Further work is, therefore, needed to determine if the pitch due to this flap deflection does cause adverse effects.

CHAPTER VIII

CONCLUSIONS

The investigation indicates that a normal acceleration controlled flap can be used effectively as a gust alleviator on present and future designs of large high speed transport type airplanes. The higher the value of static alleviation, the more effective a normal acceleration controlled flap is as a gust alleviator. The decrease in the dynamic alleviation due to a decrease in forward speed is not important, because the loads imposed on an airplane by a constant gust shape and size are more critical at the higher airplane speeds. If the natural frequency of the gust alleviator is changed by changing the spring constant and the driving hinge moment to keep the static deflection constant, the alleviator becomes more effective, as the natural frequency is increased. Changing the static margin of longitudinal stability has a negligible effect on the dynamic alleviation caused by the normal acceleration controlled flap. For a single gust, the pitch due to flap deflection is negligible when the gradient distance issmall and does not cause adverse effects when the gradient distance is large. Further work is necessary to determine whether the pitch due to flap deflection produces adverse effects in repeated gusts, OR SUBSEQUENT MOTIONS.

SYMBOLS

The following symbols are commonly used in this paper:

S	Wing area, square feet
c	wing mean aerodynamic chord, feet
$\left(\frac{dC_L}{d\alpha}\right)_w$	slope of the wing lift curve, per radian
S_t	tail area, square feet
S_{wf}	area of wing containing flap, square feet
$\left(\frac{dC_L}{d\alpha}\right)_w$	slope of the tail lift curve, per radian
W	weight of airplane, pounds
l_w	wing moment arm, feet
l_t	tail moment arm, feet
$I_{c.g.}$	pitching moment of inertia of airplane, slug-ft^2
I_A	moment of inertia of flaps, servo, and links, slug-feet^2
B	transformed unsteady-lift function, C_L, for wing, equals $(1 - C_{L_{\alpha_w}})$
C	transformed unsteady-lift function, C_L, for tail, equals $(1 - C_{L_{\alpha_t}})$
U	gust velocity, feet per second
V	forward velocity, feet per second
ρ	mass density of air, slugs per cubic foot
g	acceleration due to gravity, feet per second per second
s_1	nondimensional distance penetrated into gust, chords
s	variable of integration
Δs	increment of s used in matrix solution
Δn	load-factor increment encountered by airplane, multiples of acceleration due to gravity

θ	angle of pitch measured from initial flight path, radians
δ	flap deflection, radians
l_{w_t}	distance from trailing edge of wing to leading edge of tail, feet
l_{c_L}	lag for unsteady-lift function due to downwash to be effective on tail when wing undergoes a unit jump of angle of attack
η	tail efficiency factor
$\frac{d\epsilon_1}{d\alpha}$	asymptotic value of downwash angle per unit jump of angle on wing including effects due to unsteady lift on tail
$\left(\frac{\partial C_L}{\partial D\delta}\right)_A$	part of additional lift due to angular velocity of flap caused by acceleration of potential flow
$\left(\frac{\partial C_L}{\partial D\delta}\right)_B$	part of additional lift due to angular velocity of flap caused by effective increase in camber
k_s	servo spring constant
λ	servo damping constant
$\frac{dH_D}{d\Delta n}$	driving hinge moment per g
$\frac{dC_m}{d\delta}$	pitching moment coefficient per radian of flap deflection
$\frac{d\alpha}{d\delta}$	flap effectiveness factor, radian per radian
$C_{L_{g_w}}$	normalized unsteady-lift function due to penetration of a sharp edge gust for wing
$C_{L_{g_t}}$	normalized unsteady-lift function due to penetration of a sharp edge gust for tail
$C_{L_{\epsilon_g}}$	equivalent normalized unsteady-lift function on tail due to downwash caused by wing penetration into a sharp edge gust
$C_{L_{\alpha_w}}$	normalized unsteady-lift function for a unit jump of angle of attack for wing

$C_{L_{\alpha_t}}$ normalized unsteady-lift function for a unit jump of angle of attack for tail

$C_{L_{e_t}}$ equivalent normalized unsteady-lift function on tail due to downwash caused by a unit jump of angle of attack on wing

A_s static alleviation, percent

H hinge moment, foot-pounds

$\frac{c}{c_c}$ damping ratio, fractional part of critical damping

f natural frequency of flap, cycles per second

$\frac{dC_m}{dC_L}$ static margin, chords

Operational Symbols:

D, D^2 differential operators

I^y single integration of arbitrary function y(s)

II^y double integration of arbitrary function y(s)

Matrix Symbols:

E, F, G constant elements in matrix solution

$\begin{Bmatrix} u_1 \\ u_2 \\ 0 \end{Bmatrix}$ column matrix representing the forcing function due to gust in recurrence formula

$[M]$ square matrix representing inverse of the matrix coefficients of Δn, $D^2\theta$, and D^2 at point m

$[S_1], \ldots [S_6]$ rectangular matrices representing constant elements in recurrence formula

Subscripts:

f flap

w wing

t tail

c.g. center of gravity of airplane

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APPENDIX

Appendix A

The following are the algebraic expressions for the constants involved in the equations of motion used in this investigation:

$$K_1 = \left(\frac{dC_L}{d\alpha}\right)_w \frac{\rho}{2} V S \frac{U_{max}}{W}$$

$$K_2 = K_1 \frac{\left(\frac{dC_L}{d\alpha}\right)_t}{\left(\frac{dC_L}{d\alpha}\right)_w} \eta \frac{S_t}{S}$$

$$K_3 = -K_2 \frac{d\epsilon_1}{d\alpha}$$

$$K_4 = \left(\frac{dC_L}{d\alpha}\right)_w \frac{\rho}{2} \frac{S}{W} c g$$

$$K_5 = K_4 \frac{\left(\frac{dC_L}{d\alpha}\right)_t}{\left(\frac{dC_L}{d\alpha}\right)_w} \eta \frac{S_t}{S}$$

$$K_6 = -K_5 \frac{d\epsilon_1}{d\alpha}$$

$$K_7 = \left(\frac{dC_L}{d\alpha}\right)_w \frac{S}{W} \frac{\rho}{2} V^2$$

$$K_8 = K_7 \frac{\left(\frac{dC_L}{d\alpha}\right)_t}{\left(\frac{dC_L}{d\alpha}\right)_w} \eta \frac{S_t}{S}$$

$$K_9 = -K_8 \frac{d\epsilon_1}{d\alpha}$$

$$K_{10} = \left(\frac{dC_L}{d\alpha}\right)_w \frac{S}{W} \frac{\rho}{2} V^2 \frac{l_w}{c}$$

$$K_{11} = \left(\frac{dC_L}{d\alpha}\right)_t \eta \frac{S_t}{W} \frac{\rho}{2} V^2 \frac{l_t}{c}$$

$$K_{12} = -\left(\frac{dC_L}{d\alpha}\right)_t \eta \frac{S_t}{W} \frac{\rho}{2} V^2 \frac{l_w}{c} \frac{d\epsilon_1}{d\alpha}$$

$$K_{25} = \frac{W l_w c^2}{I_{c.g.} V^2}$$

$$K_{26} = \frac{W l_t c^2}{I_{c.g.} V^2}$$

$$K_a = \frac{\rho}{2} \frac{S}{W} V^2 \left(\frac{dC_L}{d\alpha}\right)_w \frac{d\alpha}{d\delta}$$

$$K_b = \frac{\rho}{2} \frac{S_{wf}}{W} \frac{V^2}{2} \left[\left(\frac{\partial C_L}{\partial D\delta}\right)_A + \left(\frac{\partial C_L}{\partial D\delta}\right)_B \left(\frac{dC_L}{d\alpha}\right)_w\right]$$

$$K_e = \frac{\rho}{2} S \frac{c^3}{I_{c.g.}} \frac{dC_m}{d\delta}$$

$$K_f = \frac{c^2}{I_A V^2} \frac{dH_e}{d\Delta\eta}$$

$$K_g = \frac{c^2}{I_A V^2} K_5$$

$$K_h = \frac{c}{I_A V} \Lambda$$

Appendix B

Numerical Integrating Formulas[1]

Duhamel's integral:

$$\int_0^m C_{L_\alpha}(s_1-s)\,\gamma(s)\,ds = I^{\gamma(s)}_{m-1} + \Delta s\left[\frac{9}{24}\ \frac{19}{24}\ -\frac{5}{24}\ \frac{1}{24}\right]\begin{Bmatrix}\gamma_m\\ \gamma_{m-1}\\ \gamma_{m-2}\\ \gamma_{m-3}\end{Bmatrix} -$$

$$\Delta s\left[\frac{1}{3}B_0 \text{ or } C_0\ \ \frac{4}{3}B_1 \text{ or } C_1\ \ \frac{2}{3}B_2 \text{ or } C_2\ \ \frac{4}{3}B_3 \text{ or } C_3\ \ \frac{2}{3}B_4 \text{ or } C_4\ \ \frac{4}{3}B_5 \text{ or } C_5\ \ \frac{1}{3}B_6 \text{ or } C_6\right]\begin{Bmatrix}\gamma_m\\ \gamma_{m-1}\\ \gamma_{m-2}\\ \gamma_{m-3}\\ \gamma_{m-4}\\ \gamma_{m-5}\\ \gamma_{m-6}\end{Bmatrix}$$

where

$B(s) = 1 - C_{L_{\alpha_w}}(s)$ AND

$C(s) = 1 - C_{L_{\alpha_t}}(s)$

and also

$$\int_0^{s_1} C_{L_{\epsilon_t}}(s_1-s)\,\gamma(s)\,ds = -0.158\int_0^{s_1}\gamma(s)\,ds + 1.158\int_0^{s_1-\left(\frac{l_{w_c}}{2}+0.80\right)}\gamma(s)\,ds$$

Ordinary type of integrals:

$$\int_0^m \gamma(s)\,ds = I^{\gamma}_m = I^{\gamma}_{m-1} + \Delta s\left[\frac{9}{24}\ \frac{19}{24}\ -\frac{5}{24}\ \frac{1}{24}\right]\begin{Bmatrix}\gamma_m\\ \gamma_{m-1}\\ \gamma_{m-2}\\ \gamma_{m-3}\end{Bmatrix}$$

$$\int_0^m\int_0^m \gamma(s)\,ds\,ds = II^{\gamma}_m = II^{\gamma}_{m-1} + (\Delta s)^2\frac{9}{24}\left[\frac{9}{24}\ \frac{19}{24}\ -\frac{5}{24}\ \frac{1}{24}\right]\begin{Bmatrix}\gamma_m\\ \gamma_{m-1}\\ \gamma_{m-2}\\ \gamma_{m-3}\end{Bmatrix} + \Delta s\left[\frac{7}{6}\ -\frac{5}{24}\ \frac{1}{24}\right]\begin{Bmatrix}I^{\gamma}_{m-1}\\ I^{\gamma}_{m-2}\\ I^{\gamma}_{m-3}\end{Bmatrix}$$

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1. Mazelsky and Diederich, Op. Cit., Appendix A.

Appendix C

The integral equations of motion take the following form when put into recurrance difference form for numerical solution:

$$(1)\quad \begin{Bmatrix}\Delta\eta\\ D^2\theta\\ D^2\delta\end{Bmatrix}_m = [M]\left\{\begin{Bmatrix}U_1\\ U_2\\ 0\end{Bmatrix}_m + [S_1]\begin{Bmatrix}\Delta\eta\\ D^2\theta\\ D^2\delta\\ I^{\Delta\eta}\\ I^{D^2\theta}\\ I^{D^2\delta}\\ \Pi^{D^2\theta}\\ \Pi^{D^2\delta}\end{Bmatrix}_{m-1} + [S_2]\begin{Bmatrix}\Delta\eta\\ D^2\theta\\ D^2\delta\\ I^{D^2\theta}\\ I^{D^2\delta}\end{Bmatrix}_{m-2} + \cdots\cdots + [S_6]\begin{Bmatrix}\Delta\eta\\ D^2\theta\\ D^2\delta\\ I^{D^2\theta}\\ I^{D^2\delta}\end{Bmatrix}_{m-6}\right\}$$

where

$$(2)\quad [M] = \begin{bmatrix}M_1 & M_2 & M_3\\ M_4 & M_5 & M_6\\ M_7 & M_8 & M_9\end{bmatrix} \quad \text{IN WHICH}$$

$$M_1 = \frac{F_2 G_b}{N},$$

$$M_2 = -\frac{E_2 G_b}{N},$$

$$M_3 = \frac{E_2 F_a - F_2 E_a}{N},$$

$$M_4 = \frac{G_a F_a - F_1 G_b}{N},$$

$$M_5 = \frac{E_1 G_b - E_a G_a}{N},$$

$$M_6 = \frac{F_1 E_a - E_1 F_a}{N},$$

$$M_7 = -\frac{G_a F_2}{N},$$

$$M_8 = \frac{E_2 G_a}{N}, \text{ AND}$$

$$M_9 = \frac{E_1 F_2 - F_1 E_2}{N}$$

where

$$N = E_1 F_2 G_b + G_a E_2 F_a - G_a F_2 E_a - F_1 E_2 G_b;$$

$$(3) \quad \begin{Bmatrix} u_1 \\ u_2 \\ 0 \end{Bmatrix}_m = \begin{Bmatrix} K_1 \phi_w \\ K_1 K_{26} \phi_w \\ 0 \end{Bmatrix}_m + \begin{Bmatrix} K_3 \phi_{ew} \\ -K_3 K_{26} \phi_{ew} \\ 0 \end{Bmatrix}_m + \begin{Bmatrix} K_2 \phi_t \\ -K_2 K_{26} \phi_t \\ 0 \end{Bmatrix}_{m-2}$$

IN WHICH

$$\phi_w(s) = \int_0^{s_1} C_{L_{3_w}}(s_1 - s)\, D \frac{u}{u_{MAX}}\, ds,$$

$$\phi_t(s) = \int_0^{s_1} C_{L_{3_t}}(s_1 - s)\, D \frac{u}{u_{MAX}}\, ds, \text{ AND}$$

$$\phi_{ew} = \int_0^{s_1} C_{L_{6_3}}(s_1 - s)\, D \frac{u}{u_{MAX}}\, ds;$$

AND ALSO

$$(4) \quad [S_1] = \begin{bmatrix} E_{18} & E_{24} & E_b & E_4 & E_5 & E_h & E_{11} & E_n \\ F_{18} & F_{24} & F_b & F_4 & F_5 & F_h & F_{11} & F_n \\ 0 & 0 & G_c & 0 & 0 & G_f & 0 & G_i \end{bmatrix}$$

$$(5) \quad [S_2] = \begin{bmatrix} E_{19} & E_{25} & E_c & E_6 & E_i \\ F_{19} & F_{25} & F_c & F_6 & F_i \\ 0 & 0 & G_d & 0 & G_g \end{bmatrix}$$

(6) $[S_3] = \begin{bmatrix} E_{20} & E_{26} & E_d & E_7 & E_j \\ F_{20} & F_{26} & F_d & F_7 & F_j \\ 0 & 0 & G_e & 0 & G_h \end{bmatrix}$

(7) $[S_4] = \begin{bmatrix} E_{21} & E_{27} & E_e & E_8 & E_k \\ F_{21} & F_{27} & F_e & F_8 & F_k \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(8) $[S_5] = \begin{bmatrix} E_{22} & E_{28} & E_f & E_9 & E_l \\ F_{22} & F_{28} & F_f & F_9 & F_l \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(9) $[S_6] = \begin{bmatrix} E_{23} & E_{29} & E_g & E_{10} & E_m \\ F_{23} & F_{29} & F_g & F_{10} & F_m \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ IN WHICH

$$E_1 = 1 + \frac{9}{24}\Delta S\left[K_4 - 0.158K_6 + (1.158K_6)_{m-2c_2} + K_5\right] - \frac{1}{3}\Delta S(K_4 B_0 + K_5 C_0)$$

$$E_2 = \left(\frac{9}{24}\Delta S\right)^2\left[-K_7 - K_8 + 0.158K_9 - (1.158K_9)_{m-2c_2}\right] + \frac{9}{24}\Delta S\left[K_7\,\Delta S\,\frac{B_0}{3} + K_8\,\Delta S\,\frac{C_0}{3} + K_{10} - K_{11} - 0.158K_{12} + (1.158K_{12})_{m-2c_2}\right] + \frac{1}{3}\Delta S\left(-K_{10}B_0 + K_{11}C_0\right)$$

$$E_4 = -K_4 - K_5 + 0.158K_6 - (1.158K_6)_{m-2c_2-1}$$

$$E_5 = \frac{7}{6}\Delta S\left[K_7 + K_8 - 0.158K_9 + (1.158K_9)_{m-2c_2-1}\right] - \frac{1}{3}\Delta S\left(K_7 B_0 + 4K_7 B_1 + K_8 C_0 + 4K_8 C_1\right) - K_{10} + K_{11} + 0.158K_{12} - (1.158K_{12})_{m-2c_2-1}$$

$$E_6 = \frac{5}{24}\Delta S\left[-K_7 - K_8 + 0.158K_9 - (1.158K_9)_{m-z_c-2}\right] - \frac{2}{3}\Delta S(K_7 B_2 + K_8 C_2)$$

$$E_7 = \frac{1}{24}\Delta S\left[K_7 + K_8 - 0.158K_9 + (1.158K_9)_{m-z_c-3}\right] - \frac{4}{3}\Delta S(K_7 B_3 + K_8 C_3)$$

$$E_8 = \frac{2}{3}\Delta S(-K_7 B_4 - K_8 C_4)$$

$$E_9 = \frac{4}{3}\Delta S(-K_7 B_5 - K_8 C_5)$$

$$E_{10} = \frac{1}{3}\Delta S(-K_7 B_6 - K_8 C_6)$$

$$E_{11} = K_7 + K_8 - 0.158K_9 + (1.158K_9)_{m-z_c-1}$$

$$E_{18} = \frac{19}{24}\Delta S\left[-K_4 - K_5 + 0.158K_6 - (1.158K_6)_{m-z_c-1}\right] + \frac{4}{3}\Delta S(K_4 B_1 + K_5 C_1)$$

$$E_{19} = \frac{5}{24}\Delta S\left[K_4 + K_5 - 0.158K_6 + (1.158K_6)_{m-z_c-2}\right] + \frac{2}{3}\Delta S(K_4 B_2 + K_5 C_2)$$

$$E_{20} = \frac{1}{24}\Delta S\left[-K_4 - K_5 + 0.158K_6 - (1.158K_6)_{m-z_c-3}\right] + \frac{4}{3}\Delta S(K_4 B_3 + K_5 C_3)$$

$$E_{21} = \frac{2}{3}\Delta S(K_4 B_4 + K_5 C_4)$$

$$E_{22} = \frac{4}{3}\Delta S(K_4 B_5 + K_5 C_5)$$

$$E_{23} = \frac{1}{3}\Delta S(K_4 B_6 + K_5 C_6)$$

$$E_{24} = \frac{19}{24}(\Delta S)^2\left[K_7\frac{9}{24} - K_7\frac{B_0}{3} + K_8\frac{9}{24} - K_8\frac{C_0}{3} - 0.158K_9\frac{9}{24} + \left(1.158K_9\frac{9}{24}\right)_{m-z_c-1}\right] + \Delta S\left[-K_{10}\frac{19}{24} + K_{10}\frac{4}{3}B_1 + K_{11}\frac{19}{24} - K_{11}\frac{4}{3}C_1 + 0.158K_{12}\frac{9}{24} - \left(1.158K_{12}\frac{19}{24}\right)_{m-z_c-1}\right]$$

$$E_{25} = \frac{5}{24}(\Delta s)^2\left[-K_7\frac{9}{24} + K_7\frac{B_0}{3} - K_8\frac{9}{24} + K_8\frac{C_0}{3} + 0.158K_9\frac{9}{24} - (1.158K_9\frac{9}{24})_{m-2c_c-2}\right]$$
$$+\Delta s\left[K_{10}\frac{5}{24} + K_{10}\frac{2}{3}B_2 - K_{11}\frac{5}{24} - K_{11}\frac{2}{3}C_2 - 0.158K_{12}\frac{5}{24} + (1.158K_{12}\frac{5}{24})_{m-2c_c-2}\right]$$

$$E_{26} = \frac{1}{24}(\Delta s)^2\left[K_7\frac{9}{24} - K_7\frac{B_0}{3} + K_8\frac{9}{24} - K_8\frac{C_0}{3} - 0.158K_9\frac{9}{24} + (1.158K_9\frac{9}{24})_{m-2c_c-3}\right]$$
$$+\Delta s\left[-K_{10}\frac{1}{24} + K_{10}\frac{4}{3}B_3 + K_{11}\frac{1}{24} - K_{11}\frac{4}{3}C_3 + 0.158K_{12}\frac{1}{24} - (1.158K_{12}\frac{1}{24})_{m-2c_c-3}\right]$$

$$E_{27} = \frac{2}{3}\Delta s(K_{10}B_4 - K_{11}C_4)$$

$$E_{28} = \frac{4}{3}\Delta s(K_{10}B_5 - K_{11}C_5)$$

$$E_{29} = \frac{1}{3}\Delta s(K_{10}B_6 - K_{11}C_6)$$

$$E_a = \frac{9}{24}(\Delta s)^2\left[-K_a\frac{9}{24} + K_a\frac{B_0}{3}\right] - \frac{9}{24}\Delta s K_b + K_b\Delta s\frac{B_0}{3}$$

$$E_b = \frac{19}{24}(\Delta s)^2\left[K_a\frac{9}{24} - K_a\frac{B_0}{3}\right] + \frac{19}{24}\Delta s K_b - K_b\Delta s\frac{4}{3}B_1$$

$$E_c = \frac{5}{24}(\Delta s)^2\left[-K_a\frac{9}{24} + K_a\frac{B_0}{3}\right] - \frac{5}{24}\Delta s K_b - K_b\Delta s\frac{2}{3}B_2$$

$$E_d = \frac{1}{24}(\Delta s)^2\left[K_a\frac{9}{24} - K_a\frac{B_0}{3}\right] + \frac{1}{24}\Delta s K_b - K_b\Delta s\frac{4}{3}B_3$$

$$E_e = -K_b\Delta s\frac{2}{3}B_4$$

$$E_f = -K_b\Delta s\frac{4}{3}B_5$$

$$E_g = -K_b\Delta s\frac{1}{3}B_6$$

$$E_h = \Delta s K_a\left[\frac{2}{6} - \frac{B_0}{3} - \frac{4}{3}B_1\right] + K_b$$

$$E_i = -\frac{5}{24}\Delta s K_a - K_a\Delta s\frac{2}{3}B_2$$

$$E_j = \frac{1}{24}\Delta s K_a - K_a\Delta s\frac{4}{3}B_3$$

$$E_k = -K_a\Delta s\frac{2}{3}B_4$$

$$E_l = -K_a\Delta s\frac{4}{3}B_5$$

$$E_m = -K_a \Delta S \tfrac{1}{3} B_6$$

$$E_n = K_a$$

$$F_1 = \tfrac{9}{24}\Delta S\left[K_4 K_{25} - K_5 K_{26} + 0.158 K_6 K_{26} - (1.158 K_6 K_{26})_{m-2_{CL}}\right] - \tfrac{1}{3}\Delta S\left(K_4 K_{25} B_0 - K_5 K_{26} C_0\right)$$

$$F_2 = 1 + \left(\tfrac{9}{24}\Delta S\right)^2\left[-K_7 K_{25} + K_8 K_{26} - 0.158 K_9 K_{26} + (1.158 K_9 K_{26})_{m-2_{CL}}\right] + \tfrac{9}{24}\Delta S\left[K_7 K_{25}\Delta S\tfrac{B_0}{3} + K_{10} K_{25} - K_8 K_{26}\Delta S\tfrac{C_0}{3} + K_{11} K_{26} + 0.158 K_{12} K_{26} - (1.158 K_{12} K_{26})_{m-2_{CL}}\right] + \tfrac{1}{3}\Delta S\left(-K_{10} K_{25} B_0 - K_{11} K_{26} C_0\right)$$

$$F_4 = -K_4 K_{25} + K_5 K_{26} - 0.158 K_6 K_{26} + (1.158 K_6 K_{26})_{m-2_{CL}-1}$$

$$F_5 = \tfrac{7}{6}\Delta S\left[K_7 K_{25} - K_8 K_{26} + 0.158 K_9 K_{26} - (1.158 K_9 K_{26})_{m-2_{CL}-1}\right] + \tfrac{1}{3}\Delta S\left(-K_7 K_{25} B_0 - 4 K_7 K_{25} B_1 + K_8 K_{26} C_0 + 4 K_8 K_{26} C_1\right) - K_{10} K_{25} - K_{11} K_{26} - 0.158 K_{12} K_{26} + (1.158 K_{12} K_{26})_{m-2_{CL}-1}$$

$$F_6 = \tfrac{5}{24}\Delta S\left[-K_7 K_{25} + K_8 K_{26} - 0.158 K_9 K_{26} + (1.158 K_9 K_{26})_{m-2_{CL}-2}\right] + \tfrac{2}{3}\Delta S\left(-K_7 K_{25} B_2 + K_8 K_{26} C_2\right)$$

$$F_7 = \tfrac{1}{24}\Delta S\left[K_7 K_{25} - K_8 K_{26} + 0.158 K_9 K_{26} - (1.158 K_9 K_{26})_{m-2_{CL}-3}\right] - \tfrac{4}{3}\Delta S\left(K_7 K_{25} B_3 - K_8 K_{26} C_3\right)$$

$$F_8 = \tfrac{7}{3}\Delta S\left(-K_7 K_{25} B_4 + K_8 K_{26} C_4\right)$$

$$F_9 = \tfrac{4}{3}\Delta S\left(-K_7 K_{25} B_5 + K_8 K_{26} C_5\right)$$

$$F_{10} = \tfrac{1}{3}\Delta S\left(-K_7 K_{25} B_6 + K_8 K_{26} C_6\right)$$

$$F_{11} = K_7 K_{25} - K_8 K_{26} + 0.158 K_9 K_{26} - (1.158 K_9 K_{26})_{m-2_{CL}-1}$$

$$F_{18} = \tfrac{19}{24}\Delta S\left[-K_4 K_{25} + K_5 K_{26} - 0.158 K_6 K_{26} + (1.158 K_6 K_{26})_{m-2_{CL}-1}\right] + \tfrac{4}{3}\Delta S\left(K_4 K_{25} B_1 - K_5 K_{26} C_1\right)$$

$$F_{19} = \frac{5}{24}\Delta S\left[K_4 K_{25} + 0.158 K_6 K_{26} - (1.158 K_6 K_{26})_{m-2_{cc}-2} - K_5 K_{26}\right] + \frac{2}{3}\Delta S\left(K_4 K_{25} B_2 - K_5 K_{26} C_2\right)$$

$$F_{20} = \frac{1}{24}\Delta S\left[-K_4 K_{25} + K_5 K_{26} - 0.158 K_6 K_{26} + (1.158 K_6 K_{26})_{m-2_{cc}-3}\right] + \frac{4}{3}\Delta S\left(K_4 K_{25} B_3 - K_5 K_{26} C_3\right)$$

$$F_{21} = \frac{2}{3}\Delta S\left(K_4 K_{25} B_4 - K_5 K_{26} C_4\right)$$

$$F_{22} = \frac{4}{3}\Delta S\left(K_4 K_{25} B_5 - K_5 K_{26} C_5\right)$$

$$F_{23} = \frac{1}{3}\Delta S\left(K_4 K_{25} B_6 - K_5 K_{26} C_6\right)$$

$$F_{24} = \frac{19}{24}(\Delta S)^2\left[K_7 K_{25}\frac{9}{24} - K_7 K_{25}\frac{B_0}{3} - K_8 K_{26}\frac{9}{24} + K_8 K_{26}\frac{C_0}{3} + 0.158 K_9 K_{26}\,{}^{9}/_{24} - (1.158 K_9 K_{26}\frac{9}{24})_{m-2_{cc}-1}\right] + \Delta S\left[-K_{10} K_{25}\,{}^{19}/_{24} + K_{10} K_{25}\frac{4}{3}B_1 - K_{11} K_{26}\,{}^{19}/_{24} + K_{11} K_{26}\frac{4}{3}C_1 - 0.158 K_{12} K_{26}\,{}^{19}/_{24} + (1.158 K_{12} K_{26}\,{}^{19}/_{24})_{m-2_{cc}-1}\right]$$

$$F_{25} = \frac{5}{24}(\Delta S)^2\left[-K_7 K_{25}\frac{9}{24} + K_7 K_{25}\frac{B_0}{3} + K_8 K_{26}\frac{9}{24} - K_8 K_{26}\frac{C_0}{3} - 0.158 K_9 K_{26}\,{}^{9}/_{24} + (1.158 K_9 K_{26}\frac{9}{24})_{m-2_{cc}-2}\right] + \Delta S\left[K_{10} K_{25}\,{}^{5}/_{24} + K_{10} K_{25}\,{}^{2}/_{3} B_2 + K_{11} K_{26}\,{}^{5}/_{24} + K_{11} K_{26}\frac{2}{3}C_2 + 0.158 K_{12} K_{26}\,{}^{5}/_{24} - (1.158 K_{12} K_{26}\,{}^{5}/_{24})_{m-2_{cc}-2}\right]$$

$$F_{26} = \frac{1}{24}(\Delta S)^2\left[K_7 K_{25}\,{}^{9}/_{24} - K_7 K_{25}\frac{B_0}{3} - K_8 K_{26}\,{}^{9}/_{24} + K_8 K_{26}\frac{C_0}{3} + 0.158 K_9 K_{26}\,{}^{9}/_{24} - (1.158 K_9 K_{26}\frac{9}{24})_{m-2_{cc}-3}\right] + \Delta S\left[-K_{10} K_{25}\,{}^{1}/_{24} + K_{10} K_{25}\,{}^{4}/_{3} B_3 - K_{11} K_{26}\,{}^{1}/_{24} + K_{11} K_{26}\,{}^{4}/_{3} C_3 - 0.158 K_{12} K_{26}\,{}^{1}/_{24} + (1.158 K_{12} K_{26}\,{}^{1}/_{24})_{m-2_{cc}-3}\right]$$

$$F_{27} = \frac{2}{3}\Delta S\left(K_{10} K_{25} B_4 + K_{11} K_{26} C_4\right)$$

$$F_{28} = \frac{4}{3}\Delta S\left(K_{10} K_{25} B_5 + K_{11} K_{26} C_5\right)$$

$$F_{29} = \frac{1}{3}\Delta S\left(K_{10} K_{25} B_6 + K_{11} K_{26} C_6\right)$$

$$F_a = \frac{9}{24}(\Delta S)^2\left(-K_a K_{25}\frac{9}{24} + K_a K_{25}\frac{B_0}{3} - \frac{9}{24}K_e\right) - \frac{9}{24}\Delta S K_b K_{25} + K_b K_{25}\Delta S\frac{B_0}{3}$$

$$F_b = \frac{19}{24}(\Delta S)^2\left(\frac{9}{24}K_e + K_a K_{25}\frac{9}{24} - K_a K_{25}\frac{B_0}{3}\right) + \frac{19}{24}\Delta S K_b K_{25} - K_b K_{25}\Delta S\frac{4}{3}B_1$$

$$F_c = \frac{5}{24}(\Delta s)^2\left(-\frac{9}{24}K_e - K_a K_{25}\frac{9}{24} + K_a K_{25}\frac{B_0}{3}\right) - K_b K_{25}\Delta s\left(\frac{5}{24} + \frac{2}{3}B_2\right)$$

$$F_d = \frac{1}{24}(\Delta s)^2\left(\frac{9}{24}K_e + K_a K_{25}\frac{9}{24} - K_a K_{25}\frac{B_0}{3}\right) + K_b K_{25}\Delta s\left(\frac{1}{24} - \frac{4}{3}B_3\right)$$

$$F_e = -K_b K_{25}\Delta s\frac{2}{3}B_4$$

$$F_f = -K_b K_{25}\Delta s\frac{4}{3}B_5$$

$$F_g = -K_b K_{25}\Delta s\frac{1}{3}B_6$$

$$F_h = \frac{2}{6}\Delta s(K_e + K_a K_{25}) + K_b K_{25} + \Delta s K_a K_{25}\left(-\frac{B_0}{3} - \frac{4}{3}B_4\right)$$

$$F_i = \frac{5}{24}\Delta s(-K_e - K_a K_{25}) - K_a K_{25}\Delta s\frac{2}{3}B_2$$

$$F_j = \frac{1}{24}\Delta s(K_e + K_a K_{25}) - K_a K_{25}\Delta s\frac{4}{3}B_3$$

$$F_k = -K_a K_{25}\Delta s\frac{2}{3}B_4$$

$$F_l = -K_a K_{25}\Delta s\frac{4}{3}B_5$$

$$F_m = -K_a K_{25}\Delta s\frac{1}{3}B_6$$

$$F_n = K_e + K_a K_{25}$$

$$G_a = -K_f$$

$$G_b = 1 - \frac{9}{24}\Delta s\left(\frac{9}{24}\Delta s K_g + K_h\right)$$

$$G_c = \frac{19}{24}\Delta s\left(\frac{9}{24}\Delta s K_g + K_h\right)$$

$$G_d = \frac{5}{24}\Delta s\left(-\frac{9}{24}\Delta s K_g - K_h\right)$$

$$G_e = \frac{1}{24}\Delta s\left(\frac{9}{24}\Delta s K_g + K_h\right)$$

$$G_f = \frac{2}{6}\Delta s K_g + K_h$$

$$G_g = -\frac{5}{24}\Delta s K_g$$

$$G_h = \frac{1}{24}\Delta s K_g$$

$$G_i = K_g$$

Appendix D

Values of the Unsteady Lift Due to Angle of Attack Change at the Intervals for Use in Recurrence Equations.

As given by modifying[1] Jones[2] work:

$C_{L_{\alpha w_0}} = 0.500$
$C_{L_{\alpha w_1}} = 0.714$
$C_{L_{\alpha w_2}} = 0.819$
$C_{L_{\alpha w_3}} = 0.867$
$C_{L_{\alpha w_4}} = 0.895$
$C_{L_{\alpha w_5}} = 0.912$
$C_{L_{\alpha w_6}} = 0.925$

$C_{L_{\alpha t_0}} = 0.500$
$C_{L_{\alpha t_1}} = 0.772$
$C_{L_{\alpha t_2}} = 0.862$
$C_{L_{\alpha t_3}} = 0.902$
$C_{L_{\alpha t_4}} = 0.923$
$C_{L_{\alpha t_5}} = 0.938$
$C_{L_{\alpha t_6}} = 0.949$

Modified[3] to use in the equations:

$B_0 = 0.500$
$B_1 = 0.286$
$B_2 = 0.181$
$B_3 = 0.133$
$B_4 = 0.105$
$B_5 = 0.088$
$B_6 = 0.075$

$C_0 = 0.500$
$C_1 = 0.228$
$C_2 = 0.138$
$C_3 = 0.098$
$C_4 = 0.077$
$C_5 = 0.062$
$C_6 = 0.051$

1. Maselsky and Diederich, Op. Cit., pp. 13-14
2. Jones, Op. Cit., Figure 9
3. Maselsky and Diederich, Op. Cit., Equation A1.

Appendix E

Hypothetical Airplane Parameters

$I_{c.g.}$ = 440,000 slug-ft^2

I_t = 7 slug-ft^2

W = 73,000 lbs

S = 1500 sq ft

c = 15.83 feet

b = 95 feet

A = 6

$\left(\frac{dC_L}{d\alpha}\right)_W$ = 4.30/radian

$\frac{d\alpha}{d\delta}$ = 0.138

S_t = 420 sq ft

c_t = 10.95 feet

b_t = 38.35 feet

A_t = 3.5

$\left(\frac{dC_L}{d\alpha}\right)_t$ = 3.64/radian

η = 0.9

$\frac{d\epsilon}{d\alpha}$ = 0.42

S_{wf} = 390 sq ft

l_w = 0.1c = 1.58 feet

l_t = 2.75 = 43.5 feet for $\frac{dC_m}{dC_L}$ = - 0.25

= 3.5 = 55.4 feet, for $\frac{dC_m}{dC_L}$ = - 0.35

= 4.29 = 67.8 feet, for $\frac{dC_m}{dC_L}$ = - 0.45

$\left(\frac{\partial C_L}{\partial D\delta}\right)_A$ = 0.46

$\left(\frac{\partial C_L}{\partial D\delta}\right)_B$ = 0.16

$\frac{dH_S}{d\dot{n}}$ = 0, A_s = 0 percent

= - 442 $\frac{ft\text{-}lb}{g}$, A_s = 50 percent

= - 885 $\frac{ft\text{-}lb}{g}$, A_s = 100 percent

= - 1327 $\frac{ft\text{-}lb}{g}$, A_s = 150 percent

= -4425 $\frac{ft\text{-}lb}{g}$, A_s = 500 percent

} f = 3 cycles/sec.

k_s = - 621 $\frac{ft\text{-}lb}{radian}$

λ = - 132 $\frac{ft\text{-}lb\text{-}sec}{radian}$

$\frac{dH_S}{d\dot{n}}$ = - 221 $\frac{ft\text{-}lb}{g}$

} A_s = 100 percent, f = 1.5 cycles-second

k_s = - 6,925 $\frac{ft - lb}{radian}$

λ = - 441 $\frac{ft\text{-}lb\text{-}sec}{radian}$

$\frac{dH_S}{d\dot{n}}$ = - 2465 $\frac{ft\text{-}lb}{g}$

} A_s = 100 percent, f = 5 cycles/second

$\kappa_{f/c}$ = 0.2

κ_f = 2.46 feet

S_f = 79.8 sq ft

Δs = 1.5

U_{max} = 30 F.P.S.

V = 220 F.P.S.
= 440 F.P.S.
= 586 F.P.S.

ρ = 0.002378 Slug/Cubic Foot

k_s = -2,488 $\frac{\text{Ft-lb}}{\text{Radian}}$

λ = -264 $\frac{\text{Ft-lb-sec}}{\text{Radian}}$

$\frac{dH_s}{d\Delta n}$ = -885 $\frac{\text{Ft-lb}}{\text{Radian}}$

A_s = 100 percent

f = 3 cycles/second

Appendix F

Unsteady Lift Functions due to Gust Penetration, for a Gradient Gust

m	s in wing chords	$\phi_{w_m}(s)$	$\phi_{t_{(n-\epsilon)}}(s)$	$\phi_{\epsilon_w}(s)$
0	0	0	0	0
1	1.5	0.017	0	-0.005927
2	3.0	0.044	0	-0.01185
3	4.5	0.0746	0.0188	0.02565
4	6.0	0.1062	0.0486	0.06315
5	7.5	0.1420	0.0808	0.10065
6	9.0	0.1732	0.1160	0.13815
7	10.5	0.2072	0.1498	0.17565
8	12.0	0.2422	0.1840	0.21315
9	13.5	0.2792	0.2212	0.25065
10	15.0	0.3150	0.2562	0.28815
11	16.5	0.3492	0.2934	0.32565
12	18.0	0.3854	0.3316	0.36315
13	19.5	0.4238	0.3682	0.40065
14	21.0	0.4568	0.4044	0.43815
15	22.5	0.4998	0.4420	0.47565
16	24.0	0.5324	0.4772	0.51315
17	25.5	0.5714	0.5162	0.55065
18	27.0	0.6082	0.5560	0.58815
19	28.5	0.6428	0.5940	0.62565
20	30.0	0.6812	0.6294	0.66315
21	31.5	0.7168	0.6692	0.70065
22	33.0	0.7536	0.7046	0.73815
23	34.5	0.7950	0.7432	0.77565
24	36.0	0.8318	0.7788	0.81315
25	37.5	0.8716	0.8176	0.85065
26	39.0	0.9080	0.8532	0.88815
27	40.5	0.9436	0.8924	0.92565

Appendix G

The Effect of Static Alleviation

$$A_s = -q \frac{S}{W}\left(\frac{dC_L}{d\alpha}\right)_w \frac{d\alpha}{d\delta}\frac{d\delta}{d\Delta n} \times 100$$

where q is dynamic pressure

$\frac{dS}{d\Delta n}$ is flap deflection per ξ of normal acceleration increment and is always negative in value to have positive alleviation

$$\frac{d\delta}{d\Delta n} = -\frac{A_s W}{qS\left(\frac{dC_L}{d\alpha}\right)_w \frac{d\alpha}{d\delta}\ 100}$$

For equilibrium the spring hinge moment must equal the deflecting (or driving) hinge moment.

$$k_s = \frac{dH}{d\delta} = \frac{dH_o}{d\Delta n}\frac{d\Delta n}{d\delta}$$

OR $$\frac{dH_o}{d\Delta n} = \frac{dH}{d\delta}\frac{d\delta}{d\Delta n}$$